Vagueness and the Conceptual Role of Definite Truth

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Hartry Field has recently taken the view that there is no reasonable prospect of giving an account of the truth-conditions for indefiniteness statements of the form ⌜it is indefinite whether *P*⌝ (or alternatively ⌜it is borderline whether *P*⌝) . He dismisses the previous attempts as circular, or as having implausible consequences. What is of interest here is not Field's criticism, but rather his own suggestion to explicate what it means for a sentence to be *believed* to be indefinite.[1] In what follows, some objections to Field's account shall be raised. I understand these objections as challenges which Field has not met yet, and of which it is not clear whether they can be met.

1. Field's epistemic puzzle about borderline vagueness and two explanatory theses

The point of departure for Field's argument is a certain puzzle concerning indefiniteness. If it is indefinite whether Alfred is bald, there is, so it would appear, no point in giving thought to the question whether he is bald. The problem of how to capture this intuition concerning vagueness in the form of an account of definite truth is especially acute if we start from a classical propositional logic for vague languages, in which case it follows, by the *law of excluded middle*, that there is a positive or a negative answer to the question of whether Alfred is bald. Starting from this, the puzzle we are faced with is this: *If it is either the case that Alfred is bald or it is the case that Alfred is not bald, how can it be misguided to give thought to the question which is the case?* On disquotational principles of truth and falsity for vague languages, from which, by classical propositional logic, bivalence follows, we can generate from the given object-linguistic puzzle the following metalinguistic version: *If 'Alfred is bald' is either true or false, how can it be misguided to give thought to the question which is the case?* I shall occasionally refer to the metalinguistic version, but each of the points made with respect to the metalinguistic version can be equally made with respect to the object-linguistic one.

According to Field, our puzzle concerning indefiniteness can be resolved by appeal to the conceptual content or *conceptual role* of indefiniteness. Taking Field's pionerring earlier work (1977) on conceptual role semantics as a guideline, one can sketch the underlying idea of conceptual roles as follows: sentence meaning is not exhausted by truth-conditional content; it includes a component of conceptual role. Conceptual roles (relative to a language) form constraints on what counts as a reasonable distribution of *degree of belief* or *personal probability* (in the language) − if not otherwise specified, 'probability' will be used here in the sense of 'personal probability' or 'degree of belief.'[2] A standard constraint on reasonable degree of belief is that logic be respected, in the sense that logical truths receive the maximal degree of belief, logical falsities the minimal degree, and what logically follows is not smaller in degree of belief than the premise (e.g. on standard propositional logic, a sentence must agree with its double negation in probability).

Field suggests that some constraint on probability for ascriptions of indefiniteness supplies sufficient means for resolving our puzzle concerning indefiniteness; and that this constraint does not derive from the logic of indefiniteness (or of the related notion of definite truth). To be more specific, Field's suggestion can be put in the form of the following theses:

(F1*) **(F1. Δ*)** [3] The conceptual role of definiteness is such that the probabilities of a sentence and of its negation jointly fix the probability of definiteness for the sentence.

(F1. ∇*) The conceptual role of indefiniteness is such that the probabilities of a sentence and of its negation jointly fix the probability of indefiniteness for the sentence.[4] (E.g. if 'Alfred is bald' is considered as potentially indefinite, that is, if 'It is indefinite whether Alfred is bald' has a positive probability *x*, then this fact is explicable simply by reference to the probabilities for 'Alfred is bald' and for its negation).

(F2*) The probabilistic features of sentences believed to be indefinite explain why if a sentence is believed indefinite, any further investigation appears pointless.

The conjunction of (F1*) and (F2*) has two far-reaching consequences. For one, the central role of the truth-conditional content of definite truth to a theory of indefiniteness is challenged. According to (F1*), if a sentence is considered as potentially indefinite, this amounts to the case that we can reasonably have beliefs in the sentence and in its negation to degrees which are structured in a distinctive way. That is to say, for considering a sentence as indefinite, it is not necessary to have an attitude towards any ascription of indefiniteness. We are inclined to understand certain intuitions concerning indefiniteness, such as the intuition in play in the puzzle concerning indefiniteness, as truths about the notion of indefiniteness. (F2*) suggests that this understanding is misguided.

The second consequence is this: Field's suggestion amounts to a revisionism about probability. From (F1*), it follows that the probabilities of a sentence and of its negation fix the probability that it is indefinite. That is, if two sentences are equally probable, and their negations are equally probable, then the probability of each sentence being indefinite is also equal. But this principle fails, if we jointly assume first, that probability has a classical structure, and second, that logic of definite is formally analogous to a standard normal modal logic for necessity.

The standard axiomatization for normal modal propositional logic for necessity is an extension of a standard classical propositional logic (i.e. tautologies plus modus ponens) by any normal set of axioms, that is any extension of the form:

(K) Tautologies + Modus Ponens + Universal Substitution +(Definitization: if $\vdash A$, then $\vdash \mathrm{D}A$) +(K-schema: $\vdash \mathrm{D}(A \supset B) \supset (\mathrm{D}A \supset \mathrm{D}B)$),

with optional extensions by axioms:

(T) (K) + (T-schema: $\vdash$ D$A \supset A$).

(S4) (T) + (S4-schema: $\vdash$ D$A \supset$ DDA).

(B) (T) + (B-schema: $\vdash A \supset$ D~D~A).

(S5) (T) + (S5-schema: : $\vdash$ ~D$A \supset$ D~DA).

On these assumptions, two sentences may be equally probable, and their negations may be equally probable, without the two sentences being equal in probability of their indefiniteness. And it is hard to see why we should add further principles to rule out such cases. On the contrary, such cases can be plausibly interpreted.

Consider any language containing an operator D for definite truth, truth-functions and two atomic sentences *A* and *B* which are both logically contingent and mutually logically independent (e.g. let *A* have the meaning of 'The coin lands heads on the next flip' and *B* the meaning of 'The coin is green'). If *A* and *B* agree in probability, they need not agree in probability of indefiniteness. E.g. *A* and *B* can receive both the value 0.5, in which case the same applies (by the classical additivity rule) to the probability of their negations, which is then $1 - 0.5 = 0.5$. At the same time, however, *A* and *B* need not agree in probability of definiteness or in probability of indefiniteness. More specifically, D*A* and D~*A* can take any values smaller than or equal to the value of *A* and the value of not-*A* respectively (the same for D*B* and for D~*B*). As a consequence, there is, for instance, a classical distribution on which *A*'s probability of definiteness (i.e. the probability of D$A \vee$ D$\sim A$) is 1 and B's probability of definiteness is 0 − and thus A's probability of indefiniteness (i.e. the probability of $\sim$DA & $\sim$D$\sim A$) is 0 and B's probability of indefiniteness is 1. For the considered example sentences, we can plausibly interpret such a value distribution as an epistemic situation in which there is utter ignorance both of whether A is the case and of whether B is the case; and at the same time, it is more likely that B is indefinite than that A is indefinite.

Field's thesis (F1*) suggests the adoption of some non-classical model of probability. So let us have a closer look at the particular model favoured by Field and the way it deviates from classical probability. The following is not a one-to-one exposition, but rather a reconstruction of Field's argument and the underlying probabilistic framework in a way which makes, on my view, the structure of the argument more transparent.[5]

2. Classical and Fieldian probability

Classical probability: Let us start with classical probability on a set of *propositions*, by which is meant the powerset of a given set of possible worlds. If we think of a set of possible worlds as the finest partition into logically possible worlds (or, to speak in Carnapian terms, as maximally consistent state descriptions) which can be made using some given language, these possible worlds need not be specific in every respect; nor do they need be metaphysically possible in the sense that they could really have obtained. Since the modal logical systems we consider here have the finite model property,[6] it suffices to consider *finite* possibility sets – we lose no generality by considering only functions defined on finite sets, unless we are interested in whether these functions should obey principles like countable additivity in which case we would need to consider infinite sets. Given a set of possible worlds, we can generate classical probability on the associated class of propositions as follows: Assign a measure $p(w)$ to each world w such that the sum of all measures is one – intuitively what is measured by $p(w)$ is the belief that commits one to world w. The probability $P(a)$ of a proposition a is then the sum of the measures of each world in that proposition. That is, intuitively speaking, classical probability of a proposition a measures belief which commits one to a.

For example, suppose our set of possible worlds is $\{w_1, w_2\}$, and the measure function p assigns 1/2 to each world. Consider the proposition $a = \{w_1\}$. We have $P(a) = 1/2$, and $P(\sim a) = 1/2$.

The class of classical probability functions can also be characterized without reference to measure assignments for possible worlds by saying: a function P on a given class of propositions is classical iff (i) P takes non-negative real numbers as values, (ii) P assigns value 1 to the universal set W of possible worlds, and (iii) P satisfies the classical rule of *additivity*, according to which the value of disjunctive propositions is the sum of the values of their disjuncts, if the disjuncts are mutually exclusive. In symbols, for all propositions a and b:

(P1) $P(a) \geq 0.$

(P2) $P(W) = 1.$

(P3) If a and b are mutually exclusive, then $\mathrm{P}(a \vee b) = \mathrm{P}(a) + \mathrm{P}(b)$ (*Rule of additivity*).

Analogously for *sentence* classes: For a given collection C of sentences (closed under truth-functional combinations) of a language, we can say that a function P* on C is classical iff for some classical probability function P on the class of propositions expressible in C, for every sentence A, $\mathrm{P}^*(A) = \mathrm{P}(a)$, where a is the propositional content of A. Understanding possible worlds as logically possible worlds, it follows that the logical equivalence of two sentences amounts to identity of their propositional content, and that the logical truth of a sentence amounts to its having the universal set as propositional content. On this assumption, the class of classical probability functions P* on C is given by:

(P1*) $\mathrm{P}^*(A) \geq 0$.

(P2*) If A is a logical truth, $\mathrm{P}^*(A) = 1$.

(P3*) If A and B are logically inconsistent, then $\mathrm{P}^*(A \vee B) = \mathrm{P}^*(A) + \mathrm{P}^*(B)$ (*Rule of additivity*).

Fieldian probability: Field's non-classical model of probability forms an extension of Glenn Shafer's model of belief functions (compare Shafer (1976, ch. 2)), which was motivated by considerations of belief revision. Given a set of possible worlds, one can generate a Shafer function S on the associated set of *propositions* by saying: assign a measure $m(a)$ to every non-empty proposition a, such that the sum of these measures is one – intuitively what is measured by $m(a)$ is belief which commits one to, *exactly*, a. For any proposition a, S(a) is then the sum of these measures of sets of worlds that are subsets of a – in Shafer's intuitive words, what is measured by the Shafer probability of a proposition a is the total belief which commits one, directly and indirectly, to a, by every b such that b implies a.[7] As a result, on Shafer's account of degree of belief, the classical principle of *additivity* is no longer sustainable.

To come back to our previous case, where the set of possible worlds is $\{w_1, w_2\}$: Let the measure function m assign 1/3 to each of the non-empty proposition. Consider the proposition $a = \{w_1\}$. We have $S(a) = m(\{w_1\}) = 1/3$. And $S(\sim a) = m(\{w_2\}) = 1/3$. So we have a case in which the classical principle of additivity is violated.

The class of Shafer functions can be also characterized without reference to basic measurement assignments to propositions, by the following calculus:[8] A function S on the powerset of a given set W of possible worlds is a Shafer function iff for all propositions $a, a_1, ..., a_n$:

(S1) $0 \leq S(a) \leq 1$.

(S2) $S(W) = 1$.

(S3) $S(\varnothing) = 0$.

(S4) $S(a_1 \vee ... \vee a_n) \geq \sum_{i} S(a_i) - \sum_{i<j} S(a_i \,\&\, a_j) + - ... + (-1)^{n+1} S(a_1 \,\&\, ... \,\&\, a_n)$.

For *sentence* classes, Shafer probability can be characterized as follows: A function S on a collection C of sentences (closed under truth-functional combination) of a language is a Shafer function iff for all sentences A, A_1, ..., A_n, B in C:

(S1*) $0 \leq S(A) \leq 1$.

(S2*) If A is a logical truth, then $S(A) = 1$.

(S3*) If A is a logical falsity, then $S(A) = 0$.

(S4*) $S(A_1 \vee ... \vee A_n) \geq \sum_{i} S(A_i) - \sum_{i<j} S(A_i \ \& \ A_j) + - ... +(-1)^{n+1} S(A_1 \ \& \ ... \ \& \ A_n)$.

(S5*) If A and B are logically equivalent, then $S(A) = S(B)$.

We can now see that classical probability is a limiting case of Shafer probability. Replace '≥' by '=,' and the resulting class of constraints will characterize the class of classical probability functions (*note:* the schema obtainable from the fourth rule by replacing '≥' by identity is a generalization of classical additivity to disjunctions with *n* disjuncts). Every classical function is a Shafer function. But not every Shafer function is a classical function, as the above counterexample to additivity shows.

How does all this relate to Field? Field's favoured model of probability is easily obtainable from the Shafer model simply by adding a further constraint, here called (*)-Constraint, which concerns the case in which the language contains an operator for definite truth:[9]

(*) $S(DA \vee DB) = S(A) + S(B) - S(A \& B)$.[10]

It follows from (*) that, for any sentence, its probability is, by the same token, the probability of its definite truth. I call this principle here Coincidence Principle. In symbols:

(CP) $S(DA) = S(A)$.

By (S5*), $S(A \& \bot) = S(\bot)$, which is 0, by (S3*). Now by (*), $S(DA \vee D\bot) = S(A) + S(\bot) - S(A \& \bot)$. Hence $S(DA \vee D\bot) = S(A)$. And by (S5*), $S(DA) = S(DA \vee D\bot)$. Hence $S(DA) = S(A)$.

The resulting model of subjective probability is stronger than the classical one in that the (*)-Constraint is not valid on the classical model.

Take a language containing an operator D, truth-functions and at least one atomic sentence, *A*. We construe a model <W, R, []> for the language, where W is a set (of 'possible worlds'), R a binary relation (of 'accessibility') on W, and [] a mapping from sentences to subsets of W. We can think of [*A*] as a set of possible worlds at which *A* is true. Take an S5-model M, on which W consists of two worlds *a* and *b*. R is given by $\{(a, a), (b, b), (a, b), (b, a)\}$, [*A*] by $\{a\}$. Assuming that the possible worlds in W are jointly exhaustive and mutually exhaustive and starting from a classical measurement function on W, the *m*-values for *a* and *b* have to sum up to 1. Let $m(a)$ be 1. For any sentence *X*, the classical probability $P(X)$ is then 1 if *X* is true at *a*, and 0 otherwise. As a result, we have a case in which $P(DA) < P(A)$: $P(A) = 1$ and $P(DA) = 0$. Now since classical probability cannot distinguish between logical equivalents, $P(DA) = P(DA \vee D\bot)$. Thus $P(DA \vee D\bot) \neq P(A) + P(\bot) - P(A \;\&\; \bot) = P(A) = 1$. That is we have a counterinstance to (*). A fortiori, we can construe the same case on weaker normal modal logics.

I shall turn in the next part of my discussion to the question whether Field can give any intuitive motivation in support of his probabilistic framework. At this stage, it suffices to have set it out. We can now state theses (F1*) and (F2*) with greater precision as follows:

(F1)

(F1. Δ) The probability that a sentence *A* is *definite* is given by the sum of the probabilities of *A* and its negation.[11]

(F1. ∇) The probability that a sentence *A* is indefinite is given by the extent to which the probabilities of *A* and its negation sum to less than 1[12] (N.B. It follows that a sentence is considered as potentially indefinite, just in case the probabilities of the sentence and of its negation sum up to less than 1, that is the case when classical additivity is violated).

(F2) The degree to which any investigation is concerning *A* is misguided (for a believer) amounts to the degree to which *A* is believed indefinite.[13] (N.B. Assuming – in agreement with Field – that a high degree of belief is a prerequisite for belief, it follows that if one believes *A* to be indefinite, then it would be misguided to a high degree for one to engage in further investigatiuon as to whether $A \vee {\sim}A$.

To clarify the extent to which Field's notion of probability is revisionist about the structure of probability: For any normal modal logic for definite truth at least as strong as T logic, Fieldian probability functions are classical just in case they treat every sentence as definite.

($\rightarrow$) For any Fieldian function S on any given language involving an operator D for definite truth, by (*), for any A, S(D$A \vee$ D${\sim}A$) = S(A) + S(${\sim}A$) − S(A & ${\sim}A$). By (S3*), S(A & ${\sim}A$) = 0. Hence S(D$A \vee$ D${\sim}A$) = S(A) + S(${\sim}A$). Furthermore, by (S2*), S($A \vee {\sim}A$) = 1. If S is classical, then by additivity, S($A \vee {\sim}A$) = S(A) + S(${\sim}A$). Thus S(D$A \vee$ D${\sim}A$) = 1. That is, S treats A as definite. By generalization, S treats every sentence as definite. Q.E.D. ($\leftarrow$) For any language involving an operator D for definite truth, take any Fieldian function S which treats every sentence as definite. That is, for any A, S(D$A \vee$ D${\sim}A$) = 1. For showing that S is classical, we need only to show that it meets the principle of additivity. That the latter holds, can be shown as follows: We have (1) S($A \vee B$) = 1 − S(${\sim}$($A \vee B$)): for by (*) and (S3*), we have S(D$A \vee$ D${\sim}A$) = S(A) + S(${\sim}A$), from which it follows that S(A) = 1 − S(${\sim}A$). That is for any sentence, the probability of falsity is equal to 1 minus the probability of truth. From this result, we obtain (1) as an instance. From (1), it follows that (2) S($A \vee B$) = 1 − S(D${\sim}A$ & D${\sim}B$): for by (S5*) and classical propositional logic, S(${\sim}$($A \vee B$)) = S(${\sim}A$ & ${\sim}B$); by the

Coincidence Principle (CP), $S(\sim A \,\&\, \sim B) = S(D(\sim A \,\&\, \sim B))$; and by (S5*) and K logic, $S(D(\sim A \,\&\, \sim B)) = S(D{\sim}A \,\&\, D{\sim}B)$. Now (3) $S(D{\sim}A \,\&\, D{\sim}B) = S(\sim A) + S(\sim B) - S(D{\sim}A \vee D{\sim}B)$: for $S(D{\sim}A \,\&\, D{\sim}B) = S(\sim A \,\&\, \sim B)$, and by (*), $S(\sim A \,\&\, \sim B) = S(\sim A) + S(\sim B) - S(D{\sim}A \vee D{\sim}B)$. From (2) and (3), we obtain (4) $S(A \vee B) = 1 - [S(\sim A) + S(\sim B) - S(D{\sim}A \vee D{\sim}B)]$. And from this, it follows that (5) $S(A \vee B) = S(A) + S(B) - S(\sim D{\sim}A \,\&\, \sim D{\sim}B)$: for by arithmetic and the proven fact that S-probabilities of truth and falsity sum up to 1, we have $1 - [S(\sim A) + S(\sim B) - S(D{\sim}A \vee D{\sim}B)] = S(A) + S(B) - S(\sim(D{\sim}A \vee D{\sim}B))$; and by (S5*) and classical propositional logic, $S(\sim(D{\sim}A \vee D{\sim}B)) = S(\sim D{\sim}A \,\&\, \sim D{\sim}B)$. From (5), it follows that (6) $S(A \vee B) \leq S(A) + S(B) - S(A \,\&\, B)$: for on T logic, (A & B) entails $(\sim D{\sim}A \,\&\, \sim D{\sim}B)$; and since on Fieldian probability, logical consequences of sentences *X* are not smaller in probability than *X*, we have $S(A \,\&\, B) \leq S(\sim D{\sim}A \,\&\, \sim D{\sim}B)$; from this and (5), (6) follows by arithmetic. On the other hand, we have, by (S4*), (7) $S(A \vee B) \geq S(A) + S(B) - S(A \,\&\, B)$. From (6) and (7), it follows that (8) $S(A \,\&\, B) = S(A) + S(B) - S(A \,\&\, B)$. And from this, in turn, it follows that (9) classical additivity is met: for if *A* and *B* are mutually exclusive, by (S3*), $S(A \,\&\, B) = 0$. By generalization, we have (10) classical additivity. Q.E.D.

That is, on Field's model of degree of belief, exactly in the everyday case in which some sentence of a language is not considered as definite, a departure from the classical calculus is necessary: not considering some sentence of our language as definite and violating the laws of classical probability are, according to this, two sides of the same coin.

For the remainder of the discussion, I want to focus on three issues: *First*, is there an independently plausible interpretation of belief which makes it natural to understand Fieldian functions as genuine probability functions? If the answer is negative, then for one, (F1) is, given that it is tenable at all, a purely technical result, lacking the purported sense of a psychological fact about partial beliefs. For another, (F2) is then hardly sustainable; for in the absence of any plausible interpretation of Fieldian functions, one can hardly make sense of the purported connection between Field-type degree assignments to indefiniteness ascriptions to sentences and facts about the extent to which further investigation as to the sentences's truth-value is misguided. *Second*, is the thesis (F1) tenable on the basis of the proposed model of probability? If the answer to the second question is negative, we have the result that even if there were a plausible interpretation for Field's probabilistic framework, a conceptual role account of both definiteness and indefiniteness of the form (F1) would not be feasible in this framework. This result would challenge Field's resolution strategy, insofar as the thesis (F1) turns out to be central to the strategy. This leads me to my *third* question: Can the thesis (F1) be replaced by a weaker thesis such that the thesis both is tenable in Field's framework and supplies the resources to resolve the puzzle concerning indefiniteness? What is suggested by the following considerations is that the first two questions indeed are to be answered in the negative, and furthermore that there is no straightforward positive answer to the third question. To begin with, it is argued that there seems to be no plausible interpretation in support of Field's model of probability. In the next step, a problem with the thesis (F1) concerning indefiniteness is highlighted which is, on my diagnosis, only remediable on an implausibly strong logic for definite truth. Finally, I turn to a difficulty with Field's resolution strategy, if (F1) is given up with respect to indefiniteness.

3. First Objection: Field's model of probability is ill-motivated

Field's proposed model of subjective probability comes down to an extension of the Shafer model. However he argues for his model by appeal to some sort of functions which are derivative from classical probability. The point of his argument is that we can make use of classical probabilistic intuitions for generating functions which are, on his view, to be regarded as *genuine* degree of belief distributions; and furthermore, that this very class of purported degree of belief functions does not fall into the class of classical functions, but into the class of Shafer functions meeting the (*)-Constraint. The particular sort of functions Field has in mind are functions Q which one can obtain from classical probability functions P on a language involving definite truth, with a logic which is at least as strong as S4, by means of the definition schema:

(#) $\mathrm{Q}(A) = \mathrm{P}(\mathrm{D}A)$.

In what follows, functions obtainable this way on S4 logic are called *Q-functions*. Q-functions have the following structural feature: For any sentence A, if A is treated as definite on a classical function P (that is if $\mathrm{P}(\mathrm{D}A \vee \mathrm{D}{\sim}A) = 1$), then the associated Q-function coincides with the classical function P in value for the sentence *A* and for its negation.

If (i) P(D*A* $\vee$ D~*A*) = 1, then, by the law of excluded middle, (ii) P(D*A* $\vee$ D~*A*) = P(*A* $\vee$ ~*A*). We have (iii) P(*A* $\vee$ ~*A*) = P(*A*) + P(~*A*). By the (T)-axiom for definite truth, we have (iv) P(D*A* $\vee$ D~*A*) = P(D*A*) + P(D~*A*). Since, by the (T)-axiom for definite truth, (v) if P(D*A*) $\neq$ P(*A*), then P(D*A*) < P(*A*), it follows from (ii)-(iv) that both P(D*A*) = P(*A*) and P(D~*A*) = P(~*A*). Hence, by (#), Q(*A*) = P(*A*) and Q(~*A*) = P(~*A*).

On the other hand, if *A* is not treated as definite on P, the associated Q-function differs from P for *A* in two regards: (1) The Q-probability for the sentence and for its negation is smaller than 1, so that the classical law of additivity is violated. (2) The Q-probabilities of both the sentence and of its negation meet the Coincidence Principle.

(1) Assume P(D*A* $\vee$ D~*A*) < 1. Since P(D*A* $\vee$ D~*A*) = P(D*A*) + P(D~*A*) and furthermore both Q(*A*) = P(D*A*) and Q(~*A*) = P(D~*A*), we have Q(*A*) + Q(~*A*) < 1. On the other hand, by classical additivity, P(*A*) + P(~*A*) = 1. (2) For any *X*, Q(*X*) = Q(D*X*). For by (#) Q(*X*) = P(D*X*), and on S4 logic, P(DD*X*) = P(D*X*). On the other hand, if P(D*A* $\vee$ D~*A*) < 1, then either P(*A*) $\neq$ P(D*A*) or P(~*A*) $\neq$ P(D~*A*). For P(*A*) + P(~*A*) = 1, but P(D*A*) + P(D~*A*) = P(D*A* $\vee$ D~*A*) < 1.

Not all Q-functions are classical. But all Q-functions are provably Fieldian functions.[14] These, then, are the technical facts. Now the crucial question is: What warrants Field's further claim that if a classical function and its associated Q-function are distinct in value for some sentence, the latter and not the former represents a genuine distribution of degree of belief? There is no such warrant to be found in Field's paper.

In support of his claim, Field gives a certain set of constraints on any genuine degree of belief function P* to be obtained from a given classical function P (for an S4 logic):[15] (1) P* takes for logical truths the value 1; (2) If $P^*(A) = 1$, then $P^*(DA) = 1$; (3) If $P(DA) = P(A)$ and $P(D{\sim}A) = P({\sim}A)$, then $P^*(A) = P(A)$ and $P^*({\sim}A) = P({\sim}A)$; (4) If either $P(DA) \neq P(A)$ or $P(D{\sim}A) \neq P({\sim}A)$, then either $P^*(A) \neq P(A)$ or $P^*({\sim}A) \neq P({\sim}A)$. Starting from an S4 logic, it turns out that Q-functions do fulfill all these constraints. Unfortunately, however, (4) is as much in need of justification as the (*)-Constraint: (4) implies that value distributions to sentences and their negations can only be considered as reasonable degrees of belief if both values meet the Coincidence Principle. But this is no less in need of justification than what follows from the (*)-Constraint: that value assignments to single sentences can only be considered as reasonable if the values meet the Coincidence Principle. I conclude: the given set of constraints fail to provide an effective argument for replacing classical probability by any different model.

To justify Field's revisionist stance about subjective probability, what is needed is a plausible interpretation of degree of belief (given by independently plausible constraints on reasonable degree of belief) which makes it natural to adopt the Fieldian model and not the classical one. Field not only does not give any such interpretation, but, as I shall argue in what follows, it is hard to see how he could do so.[16]

It can be plausibly held that what is to be considered as a distribution of degree of belief cannot depend on what distributions are *actually* to be found in some reference class of subjects – considering that what is aimed at is a normative notion which allows us to deem unreasonable certain degree distribitions. Field qualifies reasonable degrees of belief as being such that they could be chosen by some *idealized* subject (or in other words, as such that under some counterfactual idealizing circumstances, they would be chosen by a given subject). If we understand idealized subjects as individuals who satisfy rationality constraints imposed on degrees of belief by *subjective Bayesianism*, however, degrees of belief must conform to the laws of classical probability. This is the joint upshot of two things: first the subjective Bayesian account of degrees of belief (in a hypothesis h) as betting quotients (on h) considered as fair – where a betting quotient (p on h) is considered as fair iff one sees no advantage in betting one way (on h with the quotient p) rather than the other (against h with the quotient $(1 - p)$); and second so-called *Dutch-Book arguments*, according to which betting quotients p_1, ..., p_n for a set of hypotheses h_1, ..., h_n cannot be consistently considered as fair if they violate the laws of classical probability, in the sense that: if anyone were to accept these betting quotients, he would be susceptible to a sure loss contract.[17]

To illustrate the Dutch-Book argument strategy: One can describe bets between two individuals A and B as contracts whereby A pays the sum pS to B in exchange for the payment of the sum S if the hypothesis bet on (or against) is true, and 0 if it is not; p is called the *betting quotient* (on the hypothesis), and S is called the *stake*. The payoff conditions are then:

h	payoff to A	payoff to B
T	$S - pS$	$-(1 - p)S = pS - S$
F	$-pS$	$S - (1 - p)S = pS$

Now consider the sort of violation of additivity which, according to Field, obtains whenever we consider a sentence h as indefinite, that is: the case that the degrees of belief p and q for h and $\sim h$ respectively sum up to a value smaller than 1. That is $1 - p - q > 0$. Suppose A buys from B a pair of bets, a bet on h with the betting-quotient p and a bet on $\sim h$ with the betting-quotient q, where in each case, the stake S is 1 (Pound, Dollar or whatever). A's expected net gains are then:

h $\sim h$	A's net gain	B's net gain
T F	$1 - p - q > 0$	$p + q - 1 < 0$
F T	$1 - q - p > 0$	$q + p - 1 < 0$

That is A's expected net-gain is in any event positive, whereas the expected net-gain of B is conversely in any event negative, namely $p + q - 1$. What this shows is that: If we interpret Field-type distributions of degree of belief for the case of supposed indefiniteness as distributions of betting quotients, the resulting betting strategy is incoherent in the sense that it is susceptible to a sure-loss contract.

A subjective Bayesian account of degree of belief is thus no option for Field.

Field is inclined to resort to a *divide and rule strategy*, according to which Bayesianism is right on the proviso that cases in which we do not consider a sentence as definite are ignored. The motivation for this step (Field's remarks on this are fairly sketchy) seems to stem from the following line of reasoning:[18]

Bayesian constraints on rationality presuppose a situation in which for any sentence considered, it is not misguided to raise the question of whether what is said by a sentence is true or whether it is false – in the sense that there is a uniquely correct answer to this question, and at some point of time the answer will be established. Only on this presupposition can bets be more or less advantageous. As for sentences which are not regarded as definite, however, this presupposition fails and the Dutch-Book argument strategy is blocked.

Two points are to be made in reply to this:

First, the Divide-and-Rule argument does not show that it is natural to adopt *Field's* model for the case that some sentences are not regarded as definite. It leaves the crucial question unaddressed of how to interpret degree of belief for the case in which some sentences are not considered as definite. If Bayesian-type constraints on degree of belief break down if some sentences are not considered as definite, what are the constraints then – in particular, what constraints make the adoption of Field's model natural for this case?

Second, the Divide-and-Rule argument does not show that classical probability fails *exactly* if some sentences are not regarded as definite. The argument suggests, if at all, only that Bayesian constraints break down *if* some sentence is not regarded as definite. However, the argument needed for making Field's model of degree of belief plausible – according to which infringing the laws of classical probability is tantamount to not considering some sentence as definite – must show more: it must show that Bayesian-type constraints on degree of belief break down *if and only if* some sentence is not considered as definite. More specifically, what is needed is an effective argument which shows that the Divide-and-Rule strategy cannot be equally applied to undecidable sentences *in general*, whether they are considered as definite or not. If there is a natural candidate for such an argument, it is of the form: If a meaningful sentence is not considered as definite, further investigation is misguided not by undecidability, but by the non-existence of a uniquely correct answer to the question whether what is said is true or whether it is false (or in other words, there is *no fact of the matter* as to what is the case); and Bayesian constraints fail exactly for the case that in this very sense, further investigations concerning a sentence are misguided. However, there is no pre-theoretical evidence in support of this thesis: If for any meaningful sentence, what is said is true just in case it is not false, whether the sentence is considered as definite or not, in what sense should it be pointless to speculate about what is the case, if not in the sense that we have no means of deciding what is the case? It is hard to see how to respond to this question if not by appeal to some truth-conditional account of definiteness. But the latter way of arguing would collide with Field's claim (F2), according to which facts of misguidedness of speculation are explicable without reference to the truth-conditional content of indefiniteness.[19]

I come to my conclusion: The objection that bets on vague hypotheses are pointless does not undermine the Bayesian argument strategy for the very case that some sentences are not considered. Nor, a fortiori, does the objection motivate the adoption of Field's model for this case. What is still missing is a plausible account of degree of belief from which it follows that one can indeed coherently distribute one's degrees of beief in ways violating principles of classical probability. That is, thus far Field's central theses, (F1) and (F2), can only be treated as schemata: In the absence of any plausible interpretation, (F1) remains a purely technical thesis without any intelligible psychological significance. And the purported connection in (F2) between Fieldian value distributions (on sentences and their negations) and facts about the misguidedness of further speculations (as to the sentences's truth-value) remains unintelligible.

4. Second objection: (F1. ∇) is rejectable, even in Field's own terms

If we take (F1. ∇) as a statement about degrees of belief, it follows that one can regard a sentence as indefinite without having grasped the notion of indefiniteness. I argued that it remains unclear how to understand degrees of belief intuitively if not in the Bayesian way of betting quotients regarded as fair, in which case (F1) is not valid, neither with respect to definiteness nor to indefiniteness. The following argument suggests that even if there were such an interpretation, the thesis could be rejected with respect to indefiniteness in Field's own terms: (F1. ∇) does not hold, even if we start from the probabilistic framework and the logic of definite truth favoured by Field.

To begin with a positive result, (F1) does hold with respect to *definiteness*, for any logic on which contradictory sentences are logical falsities.

By (*), $S(DA \vee D{\sim}A) = S(A) + S({\sim}A) - S(A \,\&\, {\sim}A)$. And by logical falsity of $(A \,\&\, {\sim}A)$ and (S3*), $S(A \,\&\, {\sim}A) = 0$. Hence $S(DA \vee D{\sim}A) = S(A) + S({\sim}A)$.

One may be tempted to infer from (F1. Δ) to (F1. ∇). Consider the following line of reasoning:

Treating Fieldian functions as degree of belief functions, one can put (F1) concerning definiteness in other words by saying: The extent to which we *do not* consider a sentence as *definite* is given by the extent to which the sum of our degrees of belief in the sentence and in its negation do not sum up to 1. And from this, so it would appear, it follows that: The extent to which we *do* consider a sentence as *indefinite* is given by the extent to which the sum of our degrees of belief in the sentence and in its negation does not sum up to 1.

The inference is in fact sound if the following principle holds: the extent to which we *do not* consider a sentence as true gives the degree to which we *do* consider its *negation* as true. On the classical model of degree of belief, this principle is indeed valid, by additivity. We have for any classical function P: 1 –

$P(A) = P(\sim A)$. For Fieldian functions S, however, on any normal modal logic, this principle fails just in case the pair of sentences (A and $\sim A$) concerned is not treated as definite. In symbols:

$$S(DA \vee D{\sim}A) < 1 \text{ iff } 1 - S(A) \neq S({\sim}A).$$

Explanatory remark: ($\rightarrow$) Assume $S(DA \vee D{\sim}A) < 1$. By (*), then $S(A) + S({\sim}A) - S(A \,\&\, {\sim}A) < 1$. Thus, since $S(A \,\&\, {\sim}A) = 0$, by (S3*), $S(A) + S({\sim}A) < 1$. That is, $1 - S(A) \neq S({\sim}A)$. ($\leftarrow$) Assume $1 - S(A) \neq S({\sim}A)$. By (S2*), $S(A \vee {\sim}A) = 1$. And by (S4*) and (S3*), $S(A) + S({\sim}A) \leq S(A \vee {\sim}A)$. Hence $1 - S(A) > S({\sim}A)$. But since, by (*), $S(DA \vee D{\sim}A) = S(A) + S({\sim}A)$, we have then $S(DA \vee D{\sim}A) < 1$.

By substituting an ascription of definiteness for A, one can obtain from this as an instance:

$$S(D(DA \vee D{\sim}A) \vee D{\sim}(DA \vee D{\sim}A)) < 1 \text{ iff } 1 - S(DA \vee D{\sim}A) \neq S({\sim}(DA \vee D{\sim}A)),$$

which can be abbreviated by

$$S(\Delta\Delta A) < 1 \text{ iff } 1 - S(\Delta A) \neq S({\sim}\Delta A).$$

In words, what we must assume is this: *whenever* it is not considered as definite that a sentence A is definite (i.e. whenever $S(\Delta\Delta A) < 1$), *(F1) fails with respect to* *indefiniteness*; that is the degree of belief to which we do not consider A as definite (given by: $1 - S(\Delta A)$) is not equal to the extent to which we consider A as indefinite (given by: $S(\sim(\Delta A))$.

This result alone does not establish the existence of counterinstances to (F1) with respect to indefiniteness. In fact, on an *S5 logic* for definite truth, we can validate (F1) concerning indefiniteness; for on this logic, $\Delta\Delta A$ is a logical theorem. Since Fieldian functions respect logic, as a consequence, $S(\Delta\Delta A)$ must have a probability 1, so that any counterinstance to (F1) concerning indefiniteness is ruled out.

Assuming a logic which is at least as strong as T logic, it turns out that (F1) cannot be validated this way on any logic weaker than S5. For once we start from a T logic, we obtain automatically an S5 logic by adding the axiom $\Delta\Delta A$.

Suppose $(D(DA \vee D{\sim}A) \vee D{\sim}(DA \vee D{\sim}A))$ is added to T logic. It follows then on the assumption of ${\sim}DA$, by the (T)-axiom and distributivity, that $(D{\sim}A \vee (D{\sim}DA \,\&\, D{\sim}D{\sim}A))$; from this in turn, it follows, by the (T)-axiom (on which ${\sim}A$ entails ${\sim}DA$), that $(D{\sim}DA \vee D{\sim}DA)$, which is equivalent to $D{\sim}DA$. That is ${\sim}DA \supset D{\sim}DA$ is provable. Thus if $(D(DA \vee D{\sim}A) \vee D{\sim}(DA \vee D{\sim}A))$ is added to T logic, we get automatically an S5 logic.

The problem is that an S5 logic for definite truth is highly implausible. S5 does not allow us to state consistently *higher-order vagueness*, that is to state sentences of the form such as 'it is indefinite whether it is definite whether *A*' or 'it is indefinite whether it is definitely true that *A*.' The scope of what we are allowed to state on this logic is thus implausibly narrowed down. I do not take conceptual intuitions as something sacrosanct to be preserved by any explication of the concept in question. An account may turn out to be so high in explanatory value that this virtue outweighs the vice of being to some extent counterintuitive. However, *first*, the explanatory value of a Field's model of degree of belief remains purely hypothetical as long as no plausible interpretation for this model is provided. And *secondly*, the hypothetical explanatory gain would be, on my view, outweighed by the explanatory price to be paid in any case. For a logical ban on higher-order vagueness raises the problem of how our logical intuitions concerning definite truth can be mistaken to such an extent. In the light of these considerations, it is thus fair to say that an S5 logic would not make the case for (F1) more forceful, rather the contrary.

Now consider an *S4 logic*, that is the very logic favoured by Field. As with any other logical stronger than T, S4 does not recommend itself with overwhelming force. But there seems to be no comparably strong *pretheoretical* argument against S4 as against S5.[20] In this sense, S4 could be considered as a serious logical option. However, it turns out that on S4, Fieldian functions do not support the general thesis (F1) concerning indefiniteness – a fortiori, the same holds for any logic weaker than S4: the extent to which the Fieldian probabilities of a sentence and of its negation do not sum up to 1 is in some cases *unequal* to the Fieldian probability of the sentence's indefiniteness.

Example: Take a language containing definite truth D, truth-functions and atomic sentences, where the logic for definite truth is assumed to be S4. That is to say, functions obtainable from classical probability functions by means of the (#)-schema are Q-functions, which in turn are Fieldian functions. We construe models for the language as triples <W, R, []>, where W is a set (of 'possible worlds'), R a binary relation (of 'accessibility') on W, and [] a mapping from sentences to subsets of W. Take a simple S4-model M, on which W consists of three possible worlds *a*, *b* and *c*. R is given by {(*a*, *a*), (*b*, *b*), (*c*, *c*), (*a*, *b*), (*b*, *c*), (*c*, *b*), (*a*, *c*)}. Let *A* and *B* be two atomic sentences of the language, where [*A*] = {*a*, *c*} and [*B*] = {*a*}. Assuming the possible worlds in W as jointly exhaustive and mutually exclusive and starting from a classical measurement function *m* on W, the *m*-values for *a*, *b* and *c* have to sum up to 1. Let the course of values of *m* be: {(*a*, 1), (*b*, 0), (*c*, 0)}. We can obtain from this a classical probability function P on the language, by defining P(*S*) as 1 if *S* is true at *a*, and 0 otherwise. Take the associated Q-function for P. We have then the following result: Q(*A*) = 0, Q(~*A*) = 0, Q(*B*) = 0, Q(~*B*) = 0. But Q(~D*A* & ~D~*A*) = 1, and Q(~D*B* & ~D~*B*) = 0. Whereas (F1) holds for *A*, it fails for *B* with respect to indefiniteness.

Would the following sort of manoeuvre be an *ultima ratio*? The idea is to sustain *in a weaker sense* the thesis that the probability of a sentence and of its negation jointly determine the probability that the sentence is indefinite (F1) says that facts about the probability of a sentence and of its negation *strongly determine* facts about the sentence's indefiniteness, in the sense that once probabilities have been distributed over a sentence and its negation, that leaves but one way to fix the probability for the associated indefiniteness ascription (analogously for definiteness). It is hard to see in what plausible sense one could claim that facts of probability of truth and falsity determine facts of probability of indefiniteness without also claiming that: The probability of a sentence and of its negation *weakly determine* facts about the sentence's indefiniteness, in the sense that two sentences cannot differ with respect to facts about their probability of indefiniteness without differing with respect to facts about their and their negation's probability. Insofar as I am right in this view, it follows that one would need to sustain at least the following claim:

(F1') **(F1. Δ')** If A and B agree in the sum of the respective probabilities of the sentence and of its negation, then they also agree in the respective probabilities of definiteness (in symbols: if $S(A) + S({\sim}A) = S(B) + S({\sim}B)$, then $S(DA \vee D{\sim}A) = S(DB \vee D{\sim}B)$).

(F1. ∇') If two sentences, A and B, agree in the extent to which therespective probabilities of the sentence and of its negation sum up to less than 1, then they will also agree in their respective probabilities of indefiniteness (in symbols: if $1 - (S(A) + S({\sim}A)) = 1 - (S(B) + S({\sim}B))$, then $S({\sim}DA \,\&\, {\sim}D{\sim}A) = S({\sim}DB \,\&\, {\sim}D{\sim}B)$).

For seeing that this conceivable manoeuvre would fare no better, it suffices to appeal to the foregoing counterexample. That is, the Fieldian probabilities of a sentence and of its negation do not jointly fix the Fieldian probability of the sentence's indefiniteness.[21] Assuming that one cannot abandon (F1') without also abandoning any explicatory approach to probability of indefiniteness, the upshot of this is that there is no resort in weakening (F1).

Where are we now? It was not my ambition to explore the logical space of possible modifications of Field's strategy but to discuss some natural moves of modifying it in a way which captures its spirit. Two ideas seem to me characteristic of this position: for one, the idea that facts about probability of truth and falsity are basic to facts about probability of indefiniteness (which is in play in the strategy Field adopts to resolve the puzzle of indefiniteness); for another, the idea that definite truth obeys a logic at least as strong as S4 (which is in play in Field's strategy of generating Fieldian probability functions from classical probability functions). I considered various conceivable ways of modifying Field's strategy which accommodate both ideas. The result was negative: Unless one resorts to an implausibly strong logic for definite truth, (F1) is rejectable for Fieldian probability. And replacing (F1) by any plausible weakening makes no difference.

5. Doing without (F1. ∇)?

The rough idea of Field's approach to the indefiniteness puzzle is to explain why, if a sentence *A* is believed indefinite, it is pointless (relative to the believer) to speculate about whether *A* or whether ~*A*. The link between belief in indefiniteness and misguidedness of investigation is thought of as some constraint on degree of belief relating to the conceptual role of definite truth. What Field's strategy suggests is that this link can be provided by an account of degree of belief in indefiniteness in terms of associated degrees of belief in truth and in falsity. The foregoing result suggests that no such link can be provided, for no such account is feasible in a Fieldian framework of degree of belief – unless one resorts to an ill-motivated strong logic for definite truth. This leaves open the question whether Field's approach needs to rely on a conceptual role account of indefiniteness, that is anything like the thesis (F1) concerning indefiniteness. I think that the removal of (F1) concerning indefiniteness raises a problem for Field's approach, and it is unclear whether this problem can be remedied without difficulty.

If degree of belief in indefiniteness and degree of misguidedness of further investigation are connected in the way suggested in (F2), it is hard to see how to explain this connection without appeal to some distinctive feature of degree of belief in indefiniteness, given by anything like (F1) with respect to indefiniteness. In this regard, there seems to be no point in keeping to (F2), once (F1) concerning indefiniteness (or anything like it) is given up.

It is hard to see how to deal with this problem; and what I can do here is just to consider one way which seems to be the most straightforward one: The idea is that the missing link between belief in indefiniteness and misguidedness of further investigation may be provided by the characteristic features of belief in *definiteness*. More specifically, what is suggested is to replace (F2) by something like this:

(F2') The degree to which any investigation as to a sentence *A*'s truth-value is misguided amounts to the degree to which *A* is not believed to be definite, that is the extent to which the degree of belief in *A*'s definiteness differs from 1.

If a sentence is believed indefinite to some degree x, it follows (in a Fieldian framework) that it is not believed definite to a degree y, where $x \leq y$. That is (assuming that a high degree of belief is a prerequisite for belief), *if a sentence A is believed indefinite, it is highly misguided to wonder whether A is the case or whether ~A is the case*. The problem is that (F2') not only supports this plausible principle but also this one: *if we have only small confidence in the definiteness of a sentence A and at the same time no confidence at all in the indefiniteness of the sentence, it is highly misguided to wonder whether A is the case or whether ~A is the case*. There is patently no pre-theoretical evidence in support of this principle.

Conclusion: If we consider a sentence as indefinite, do we violate the laws of classical probability? The position of Field's paper that the answer is positive is open to serious objections in two regards. First it lacks the necessary theoretical underpinning in the form of a plausible interpretation of subjective probability. And second it relies on the thesis (F1. ∇) that probability distributions for sentences and their negations constrain the probability of the indefiniteness of the sentence; and this thesis can be challenged in Field's own terms – if we do not adopt an implausibly strong logic for definite truth. My final consideration suggested that the thesis is not removable from Field's strategy without difficulty.

I do not rule out the possibility of further moves which may remedy the mentioned problems. But it is still to be shown that such moves can be made.[22]

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[1] Field (2000, 293-303) and (forthcoming, 21-4).

[2] This is just a terminological convention, which does not hinder me from being neutral regarding the controversial question of whether the subjectivist view of probability is right, according to which there are no facts of probability but facts of personal probability assignments by subjects.

[3] 'Δ' and '∇' are the standard abbreviations for definiteness and indefiniteness respectively, definable in terms of definite truth.

[4] In what follows, the phrases 'considering a sentence as potentially in/definite' and 'considering a sentence as in/definite' are used in Field's intended sense of 'considering a sentence as in/definite to some positive degree' and of 'considering a sentence as in/definite to the degree 1' respectively.

[5] Stephen Schiffer's papers, (1998) and (2000), are similar to Field's account in spirit. There are however important differences between Field and Schiffer: If I understand Schiffer correctly, he is concerned with a psychological theory of the *truth-conditions* of indefiniteness. According to this, a proposition *P* is indefinite just in case we can reasonably have a distinctive *kind* of belief attitude towards the proposition *P*. Whereas precision-related beliefs have a classical probabilistic structure, on Schiffer's view, vagueness-related beliefs conform to the infinite-valued Łukasiewicz logic (at least for the fragment of propositional logic). Considering this, the similarity between Field and Schiffer is rather superficial, and the discussion of Field's account loses nothing when we put aside Schiffer's account.

[6] Hughes/Cresswell (1996, ch. 8).

[7] Shafer (1976, 38). In contrast to classical probability, it remains unclear how to understand Shafer probability as a measure of belief.

[8] Shafer (1976, 39).

[9] Compare Field (2000, 298). Field has indicated (in conversation) that the Shafer model plus (*) captures adequately the conception of degree of belief he argues for in (2000).

[10] Field does not explicitly expound his notion of degree of belief in the form of the axiomatization given here. That the latter indeed captures his notion of degree of belief has been recently confirmed by him (conversation with Field in June 2002).

[11] Field (2000, 299) and (forthcoming, 23).

[12] Field (2000, 300) and (forthcoming, 24).

[13] Field (2000, 303).

[14] Just some explanatory remarks: (i) That Q satisfies (S1*) is trivial; (ii) that it satisfies (S2*), is provable by appeal to the definitization rule saying that if A is logical truth, so is DA; (iii) that Q satisfies (S3*), can be shown as follows: if A is a logical falsity, then $\sim A$ is a logical truth, and thus, by satisfaction of (S2*), $Q(\sim A) = 1$. By (S4*), it follows then that $Q(A)$ must be 0. (iv) that Q satisfies (*) is provable by appeal to S4 logic, on which $D(DA \vee DB)$ is logically equivalent to $DA \vee DB$ (see Field (2000, p. 298)); (v) from satisfaction of (*), it follows that Q also satisfies (S4*): for by respect of logic by classical probability, $P(D(A \vee B)) \geq P(D(DA \vee DB))$, and hence, by (#),$Q(A \vee B) \geq Q(DA \vee DB)$; by (*), then $Q(A \vee B) \geq Q(A) + Q(B) - Q(A \,\&\, B)$, which can be generalized to (S4*). (vi) Q satisfies (S5*) , for: If $A \equiv B$ is a logical truth, so is $DA \equiv DB$, by definitization and distributivity. Let P be a classical function from which Q is obtainable. Since classical probability does not distinguish between logical equivalents, we have $P(DA) = P(DB)$, and thus, by (#), $Q(A) = Q(B)$.

[15] Field (2000, 294).

[16] I put aside here Shafer's attempt, in (1981, 5), at providing a more definite key to understanding his calculus in terms of a so-called set of *canonical examples*. Canonical examples are to be thought of as coded messages representing bodies of evidence. Suppose

somebody chose a code at random from a specified set and sent us the result. We know the set of codes and the chance of each code being chosen. The decoded messages are all of the form 'The true hypothesis is in A,' where A is a proposition (that is a subset of a given set of possible worlds). With probability p_i, the message (of the body of evidence) is 'The true hypothesis is in A_i.' Let $m(A)$ be the sum of all p_i such that $A_i = A$. $m(A)$ gives the total chance that the message is 'The true hypothesis is in A.' The function S on propositions, defined by $S(A) = \Sigma m(B)$, where the summation is over $B \subseteq A$, is a Shafer function. Starting from Shafer's interpretation of m, the function $S(A)$ gives then the total chance that the message implies that the true hypothesis is in A. Shafer's interpretation relies on the idea that bodies of evidence are something like randomly encoded messages sent by God or Nature. For a persuasive critique of this, see Howson/Urbach (1996, 427-8). For my purposes, it suffices to point out that Shafer's interpretation hardly provides a justification for adopting the (*)-Constraint, which is essential to Field's favoured model of degree of belief.

[17] For a clear exposition of the Dutch Book Theorem, see Howson/Urbach (1993, ch. 5). One can even argue that the strong rationality constraints of subjective Bayesianism entail perfect accuracy about one's own probabilities (compare Milne (1991)). For a less idealizing conception of epistemic probability, see Williamson (2000, esp. 209-37).

[18] Field (2000, 303).

[19] Another point, in parenthesis: The Divide-and-Rule argument also does not show conclusively that Bayesian constraints on degree of belief fail for the class of sentences for which further investigation is misguided, whether these sentences are considered as definite or not. It does not affect in any regard the Bayesian constraint on coherent distributions of betting quotients thought of as fair (according to which the net advantage in betting at the quotients involved should be zero). It only suggests that degrees of belief are to be explicated in terms of fair betting quotients relative to *idealized* betting situations in which any sentence is decidable – even if it is actually undecidable. The more general issue reflected by the particular problem of undecidability is this: Can the Bayesian give a canonical form of counterfactual betting scenarios such that any *disturbing factor* is ruled out, that is any factor which (a) may make subjects consider betting quotients as fair which patently bear no relation to their degree of belief and/or (b) may make subjects refrain from considering betting quotients as fair which accord to their degree of belief? And if so, what does this canonical form look like? The second question is contentious (see Howson/Urbach (1993, ch. 5)), and one can also harbour some doubts as to whether the first question has a positive answer at all. Notwithstanding this general doubt, however, it is far from clear that the Bayesian cannot deal with undecidability as a disturbing factor ruled out by the canonical form of betting situations considered.

[20] I have to add here in parenthesis that starting from certain *theoretical* views of definite truth as a modality and of the related accessibility relation, one can reasonably argue against the S4 axiom. If we think of accessibility as an epistemic relation of indiscriminability, it seems to be highly objectionable to treat the relation as transitive; c.f. Williamson (1990). If we think of the accessibility relation as a similarity relation, one should expect the relation to be symmetric and thus the B-axiom to hold. In this case, however, again transitivity is challenged insofar as we want to avoid an S5 logic, on which we cannot accommodate higher-order vagueness; Williamson (1994, 297, n. 33).

[21] As a consequence of this, (F1) cannot be weakened either to:

(F1'') The extent to which the Fieldian probabilities of a sentence A and of its negation sumup to less than 1, *measures the same property as* the Fieldian probability of indefiniteness for the sentence.

For (F1'') cannot be true without (F1') being true, by the following basic fact of measurement theory: *Two mappings g and h from a domain of objects into the domain of reals cannot measure the same property of objects, if for some pair of objects s and t, g(s) < g(t) and h(s) ≥*

h(t). I refer here to the standard representational approach to measurement theory; compare Krantz et al. (1971).

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